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# Personal Taxes, Endogenous Default, and Corporate Bond Yield Spreads

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**T**erm structure models have often been criticized for failing to explain satisfactorily the yield spread between corporate and Treasury bonds. A potential problem is that the personal tax effect is ignored in these models. In this paper, we employ a structural model to investigate the role of personal taxes on both debt and equity returns in capital structure decisions and assess their impact on corporate bond yield spreads. It is shown that personal taxes affect the firm's optimal capital structure, and the tax premium explains a substantial portion of yield spreads, especially for high-grade bonds. The predictive ability of the model for yield spreads is much improved when personal tax effects are accounted for. In controlling for the liquidity effect, we obtain implied personal income tax rates closely in line with Graham's (1999) estimates.

*Key words:* structural approach; endogenous default; personal taxes; yield spread; risk neutrality

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## 1. Introduction

Term structure models have often been criticized for their inability to generate high enough spreads to be consistent with the spreads observed in the corporate bond market. Since the seminal work of Black and Scholes (1973) and Merton (1974), a vast literature has been developed in an attempt to improve the performance of these models to catch up with observed spreads by allowing for stochastic interest rates, jumps, strategic defaults, coupons, and incomplete accounting information. For example, Black and Cox (1976), Kim et al. (1993), Leland (1994, 1998), Goldstein et al. (2001), Anderson and Sundaresan (1996), Anderson et al. (1996), and Mella-Barral and Perraudin (1997) consider endogenous or strategic defaults. Kim et al. (1993), Longstaff and Schwartz (1995), and Huang et al. (2003) consider stochastic interest rates. Collin-Dufresne and Goldstein (2001) incorporate stationary leverage ratios. Zhou (2001) and Huang and Huang (2003) incorporate jump risk. Duffie and Lando (2001) take into account noisy accounting information. Despite these efforts, the so-called credit spread puzzle remains alive and well today, although in a somewhat different form. Jones et al. (1984) first documented the credit spread puzzle that a simple Merton (1974) model of default risk

applied to zero-coupon bonds substantially underestimates spreads. Collin-Dufresne et al. (2001) find that structural models explain only a small portion of changes in spreads. In a comprehensive study, by calibrating a large class of term structure models to be consistent with historical default loss experience, Huang and Huang (2003) conclude that default risk accounts for only a small portion of the observed corporate-Treasury spread for investment-grade bonds. Moreover, Eom et al. (2003) show that while most structural models fail to account for major determinants of credit spreads, they can both underestimate and overestimate spreads. They find that for most models, the accuracy of prediction remains a serious problem; predicted spreads are often too small for safer bonds or too large for high-risk bonds, while the average prediction error is not very informative.

A consensus from these studies is that some major factors are still missing in the term structure models, of which a potentially important candidate is personal taxes. It has been long recognized that taxes affect yield spreads (see Kidwell and Trzcinka 1982, Trzcinka 1982, Yawitz et al. 1985, and Miller 1977). However, most term structure models have ignored personal taxes, a rather surprising omission. Although some studies have proclaimed that taxes

explain a sizable proportion of credit spreads and, in relative terms, are the most important determinant of credit spreads of safer bonds, few serious theoretical models, or empirical assessments have been attempted. Elton et al. (2001) are the first to examine the effect of personal taxes using a reduced-form term structure model. Cooper and Davydenko (2003) develop a structural model based on the Merton (1974) framework demonstrating that taxes may account for 10 to 35 basis points of corporate bond spread. In their model, tax spread is independent of default risk because of assumptions of symmetric taxation for capital gains and ordinary income, as well as the independence of capital structure from personal taxes.

In this paper, we propose a model to incorporate the effects of personal taxes associated with both debt and equity returns on the term structure of corporate bonds. To the best of our knowledge, our paper represents the first effort to quantify the relative importance of personal tax effects on debt and equity prices by taking into account their feedback into capital structure decisions. Our model is a generalization of the traditional structural models with endogenous default along the line of Leland (1994) and Leland and Toft (1996). The structural approach offers distinct advantages for modeling the effects of taxes and default. First, the structural approach provides an integrated framework that simultaneously addresses the issues related to capital structure, default probability, and yield spreads. It allows us to establish the linkage among important financial decisions and identify channels through which personal tax effects may occur. This contrasts with the reduced-form or time-series approach, which takes either spreads or default probability as exogenously given and deals with these two variables separately. Second, the so-called credit spread puzzle, or to a lesser extent, the default probability level puzzle, exists largely because of an investigation within the contingent claims framework.<sup>1</sup> Therefore, the structural approach is a natural choice to explain these puzzles. Finally, structural models appear to provide more economic insight and predictive power (see Collin-Dufresne et al. 2003), which can help us better understand the nature of the personal tax effect and its economic significance.

We chose the Leland-Toft (1996) approach to investigate the role of personal taxes because it allows us to determine debt values (and yield spreads) and optimal capital structure in a unified framework. This framework effectively deals with the issue of the

interrelationship among taxes, capital structure, and yield spreads. A complication involved in this issue is that debt values and capital structure are interdependent. Within this structural framework, default is modeled as an optimal strategy by equity holders, whereby the default boundary and capital structure are endogenously determined. The debt value depends on the default boundary optimally chosen by equity holders. It can be shown that personal taxes affect equity holders' optimal choice of capital structure and default boundary which, in turn, affects default risk, debt value, and yield spread. We find that personal taxes generally reduce the value of risky debts, and more importantly, that there are profound interactive effects among taxes, leverage, and default risk.

The key question to be addressed is how much of the observed corporate spread is because of personal taxes. To answer this question, we calibrate the model based on the historical default and equity return data as in Huang and Huang (2003). The calibration allows us to obtain estimates of yield spreads that match the observed default loss experience and market risk premium. This procedure generates robust estimates of yield spreads. Huang and Huang (2003) show that very different models produce remarkably consistent spreads when they are calibrated to fit observed default loss data. This conclusion holds even after they account for the countercyclical market risk premium and jump risk in the firm value process. A significant implication from this finding is that choice of model structure is not terribly important. Thus, one can use a particular model to draw a reliable inference rather than involve a large class of models in the literature, because the predictive ability of these models will be comparable once they are carefully calibrated to actual default data.

We use our calibrated model to assess the effects of personal taxes. The calibrated model generates a more precise quantitative measure of tax spread. We find that personal taxes account for a substantial portion of spreads for investment-grade bonds. When personal taxes are ignored, the model explains only 31%–41% of investment-grade bond spreads. After including empirical tax rates documented by Graham (1999) into the model, the percentage of the investment-grade bond spread explained by the model increases to a range of 47%–87%. The percentage increase in spread because of personal taxes is particularly significant for AAA (from 41%–87%) and AA (from 33%–65%) bonds. Furthermore, by adjusting observed spreads for liquidity premiums of bonds with different ratings (see de Jong and Driessen 2004), we obtain implied personal tax rates from observed yields. The implied tax rates for investment-grade bonds are very close to those documented by Graham (1999). Results

<sup>1</sup> The authors thank an anonymous referee for pointing out this important fact. Empirically, default probability implied by the yield spread has been much higher than historical default probability.

show that the structural model performs very well and personal taxes are an important determinant of corporate spreads. Recently, Leland (2002) has shown that structural models can predict default probability very accurately. Schaefer and Strebulaev (2004) find that structural models explain a large fraction of returns on investment-grade bonds and provide accurate predictions of hedge ratios. These findings suggest that the failure of structural models to explain corporate bond spreads is more likely because of missing factors unrelated to credit risk than to their inability to predict default probability. Our results strongly support this view by identifying personal taxes as an important missing factor unrelated to credit risk. Our findings also suggest that it will be fruitful to incorporate personal taxes into these two models to see whether their performance can be improved.

The remainder of this paper is organized as follows. Section 2 generalizes the Leland-Toft (1996) model to incorporate the effects of personal taxes. Section 3 conducts numerical analysis to examine the performance of the model and shows how personal taxes affect capital structure, default boundary, firm value, bond yields, and yield spread. Section 4 introduces the calibration procedure and presents the results generated from the calibrated model. Section 5 presents concluding remarks.

## 2. Finite Maturity Debt with Personal Taxes

### 2.1. The General Setting

In this section, we introduce personal taxes into the term structure model. The tax environment for the model is as follows.<sup>2</sup> The coupon payment of a corporate bond is taxed as ordinary income and capital gains are taxed at the capital gains tax rate, which depends on the length of time the bond is held. If the holding period is less than a year, gains are treated as short-term gains and taxed at the ordinary income tax rate. If the holding period is one year or longer, gains are taxed at a lower long-term capital gains tax rate.<sup>3</sup> If default occurs, the default loss is treated as a capital loss. The investor receives a rebate from the government, which depends on his tax status and the length of time the bond was held. The long-term capital gains (or loss) tax rate is typically a fraction of the regular income tax rate. Denote the ordinary income tax rate as  $\tau$  and the capital gains tax rate as  $\alpha\tau$ ,

where  $0 < \alpha \leq 1$ . Historically, long-term capital gains tax rates have varied from 20%–100% of the ordinary income tax rate.<sup>4</sup> Interest income and capital gains from corporate bonds are subject to both state ( $\tau_s$ ) and federal ( $\tau_f$ ) taxes. Therefore, the effective ordinary income tax rate for corporate bond investors is  $\tau = \tau_f + \tau_s(1 - \tau_f)$  because state tax is a deduction against federal income tax.

Consider that the evolution of the asset value of an unlevered firm,  $V$ , has the following continuous diffusion process:

$$\frac{dV}{V} = [\mu(V, t) - \delta]dt + \sigma dZ, \quad (1)$$

where  $\mu(V, t)$  is the total expected rate of return on the firm's assets,  $\delta$  is the total payout ratio, which is a proportion of value paid to all security holders,  $Z$  is a standard Wiener process, and  $\sigma$  is the constant volatility parameter of the asset return (see Leland and Toft 1996, Merton 1974, Black and Cox 1976, and Brennan and Schwartz 1978). The value  $V$  includes the net cash flows generated by the firm's activities and excludes cash flows from debt financing.

Next, consider a risky debt issue with maturity  $t$  periods from the present. The debt has a continuous constant coupon flow  $c(t)$  and principal  $p(t)$ . There exists a default-free asset in the market, which pays a continuous interest rate  $r$ . The value process of the levered firm continues without time limit until it reaches a default boundary  $V_B$ . Once the asset value reaches this boundary, the firm defaults on its debt. If default occurs, bondholders receive a fraction  $\rho$  of the asset value  $V_B$ , where  $\rho$  is assumed to be a constant, or  $\beta = (1 - \rho)$  is the fraction of firm value lost because of default. Bondholders pursue a buy-and-hold investment strategy.

Under the risk-neutral valuation, the value of the debt,  $d$ , is given by

$$\begin{aligned} d(V, V_B, t) = & \int_0^t (1 - \tau)c(s)e^{-rs}[1 - F(s, V, V_B)]ds \\ & + [p(t) - \alpha\tau(p(t) - d(V, V_B, t))]e^{-rt} \\ & \cdot [1 - F(t, V, V_B)] \\ & + \int_0^t [\rho V_B + \alpha\tau(d(V, V_B, s) - \rho V_B)]e^{-rs} \\ & \cdot f(s, V, V_B)ds, \end{aligned} \quad (2)$$

where  $F(s, V, V_B)$  is the cumulative default probability up to time  $s$ , and  $f(s, V, V_B)$  is the incremental default probability from time  $s$  to  $s + \Delta s$  when

<sup>2</sup> For a detailed tax treatment of fixed income securities and default loss, see Fabozzi and Nirenberg (1991), Yawitz et al. (1985), and Constantinides and Ingersoll (1984).

<sup>3</sup> The required holding period for long-term gains was six months in 1942–1977.

<sup>4</sup> Also, in some periods, alternate treatment could be elected (e.g., for large gains). Differential deductibility or tax rebate of default loss has been in place since 1922. In the Tax Reform Act of 1986, capital gains and regular income are treated equally and there is no differential tax treatment between short- and long-term gains. The top individual capital gains tax rate is currently 15%.



the drift rate is  $r - \delta$ . The first term on the right side is the discounted expected after-tax value of the coupon flow paid at time  $s$  with a probability of  $(1 - F(s, V, V_B))$ . The second term represents the discounted expected after-tax repayment value of principal, where  $\alpha\tau(p(t) - d)$  is the capital gains tax liability at maturity, with a probability of no default before maturity  $t$ ,  $(1 - F(t, V, V_B))$ . The third term is the expected residual debt value  $\rho V_B$  plus the tax rebate,  $\alpha\tau(d - \rho V_B)$ , from the investment loss if default occurs before maturity. The values of the first and second terms are negatively related to the personal income tax rate, while the third term is positively related given  $V_B$ .

By integrating (2), we can obtain the debt value

$$\begin{aligned} d(V, V_B, t) &= \frac{(1-\tau)c(t)}{r} + e^{-rt} \left[ p(t) - \alpha\tau(p(t) - d(V, V_B, t)) \right. \\ &\quad \left. - \frac{(1-\tau)c(t)}{r} \right] (1 - F(t)) + \left[ \rho(t)V_B + \alpha\tau(d(V, V_B, t) \right. \\ &\quad \left. - \rho(t)V_B) - \frac{(1-\tau)c(t)}{r} \right] G(t), \end{aligned} \quad (3)$$

where  $G(t) = \int_{s=0}^t e^{-rs} f(s; V, V_B) ds$ . Solving (3) for the debt value yields

$$\begin{aligned} d(V, V_B, t) &= \left( \frac{(1-\tau)c(t)}{r} + e^{-rt} \left[ (1-\alpha\tau)p(t) - \frac{(1-\tau)c(t)}{r} \right] \right. \\ &\quad \cdot (1 - F(t)) + \left[ (1-\alpha\tau)\rho(t)V_B - \frac{(1-\tau)c(t)}{r} \right] G(t) \Big) \\ &\quad \cdot (1 - \alpha\tau[e^{-rt}(1 - F(t)) + G(t)])^{-1}, \end{aligned} \quad (4)$$

where  $F(t)$  and  $G(t)$  are given by

$$F(t) = N[h_1(t)] + \left( \frac{V_B}{V} \right)^{2a} N[h_2(t)], \quad (5)$$

$$G(t) = \left( \frac{V_B}{V} \right)^{a-z} N[q_1(t)] + \left( \frac{V_B}{V} \right)^{a+z} N[q_1(t)]. \quad (6)$$

$N(\cdot)$  denotes the cumulative standard normal distribution, and

$$\begin{aligned} q_1(t) &= \frac{-b - z\sigma^2 t}{\sigma\sqrt{t}}, & q_2(t) &= \frac{-b + z\sigma^2 t}{\sigma\sqrt{t}}, \\ h_1(t) &= \frac{-b - a\sigma^2 t}{\sigma\sqrt{t}}, & h_2(t) &= \frac{-b + a\sigma^2 t}{\sigma\sqrt{t}}, \\ a &= \frac{r - \delta - (\sigma^2/2)}{\sigma^2}, & b &= \ln\left(\frac{V}{V_B}\right), \\ z &= \frac{[(a\sigma^2)^2 + 2r\sigma^2]^{1/2}}{\sigma^2}. \end{aligned} \quad (7)$$

The cumulative distribution function of the first passage time to default,  $F(t)$ , and the integrated discounted density of the first passage time,  $G(t)$ , are critical elements for the determination of the debt value. It is important to note that  $F(t)$  and  $G(t)$  depend on  $V_B$ , which, as will be shown later, is a function of personal taxes. Thus, personal taxes affect the default strategy of equity holders and debt value.

It is assumed that the firm continuously issues a constant principal amount of new debt with maturity  $T$  years from issuance, which will be redeemed at par upon maturity if the firm remains solvent. New debt is issued with a principal amount of  $p = P/T$  each year, where  $P$  is the total principal amount of outstanding debts. The same amount of old debt is retired per year, and so the debt structure is stationary. So long as the firm remains solvent, the total outstanding principal amount is  $P$  at any time  $s$  and the firm has a uniform distribution of principal over maturities in the interval  $(s, s + T)$ . The total coupon payment is  $C$  per year and there are  $T$  debts, each with a constant coupon payment of  $c = C/T$  and a principal amount of  $p$ . The total annual amount of debt service is equal to  $C + P/T$ . The value of all outstanding debts  $D$  can be determined by integrating the debt flow  $d(V, V_B, t)$  over the period of  $T$ :

$$D(V, V_B, T) = \int_{t=0}^T d(V, V_B, t) dt. \quad (8)$$

The value of  $D$  depends on the personal income tax rate as  $d$  is a function of  $\tau$  (see (3)).

## 2.2. Endogenous Default Boundary and the Optimal Capital Structure

The tax shield benefit of interest payments offers an incentive for firms to issue debt. But as the firm's leverage rises, the likelihood of default increases. Because default is costly, firms must weigh the tax benefit of leverage against the cost of financial distress in the debt financing decision. In addition, the corporate tax savings from debt interest service are offset by the personal tax disadvantage to bond investors. The personal tax disadvantage to investors from holding debt relative to holding equity causes them to demand higher pretax returns on corporate debt, thereby reducing the firm's incentive for debt financing. Furthermore, equity returns are also subject to taxes, with an effective rate typically lower than that on debts. The higher the taxes are on equity, the more likely it is that the firm will issue debt. The optimal capital structure thus hinges on both debt and equity taxes.

The total value of the levered firm is equal to the asset value of the unlevered firm plus leverage benefits less bankruptcy costs. Given the asset value of the unlevered firm  $V(t)$ , equity value  $E(V, V_B, T)$ , tax

benefit  $h(V, V_B)$  of leverage, bankruptcy cost  $B(V, V_B)$ , and total outstanding debt value  $D(V, V_B, T)$ , the levered firm's value  $W(V, V_B, T)$  can be written as

$$W(V, V_B, T) = V + h(V, V_B) - B(V, V_B), \quad (9)$$

where

$$h(V, V_B) = \left(1 - \frac{(1 - \tau_C)(1 - \tau_E)}{1 - \tau}\right) \frac{C}{r} \left[1 - \left(\frac{V_B}{V}\right)^{a+z}\right] + \tau_{EC} E(V, V_B, T) \left(\frac{V_B}{V}\right)^{a+z}, \quad (10)$$

$$B(V, V_B) = \beta V_B \left(\frac{V_B}{V}\right)^{a+z}, \quad (11)$$

and  $\tau_C$  is corporate income tax rate,  $\tau_E$  is the effective tax rate on equity returns, and  $\tau_{EC}$  is the capital gains tax rate on equity. Both dividends and capital gains of stock are subject to taxes. Dividend income is taxed at the ordinary income tax rate  $\tau$ , and capital gains are taxed at the capital gains rate  $\tau_{EC}$ . The effective tax rate on equity returns  $\tau_E$  is the weighted average of dividend and capital gains tax rates.

Graham (2003) suggests that the effective equity tax rate is  $\tau_E = (1 - \delta)\alpha\tau + \delta\tau$ , where the weight depends on payout ratio  $\delta$  and  $\tau_{EC} = \alpha\tau$  (Graham 2003, p. 1095; 2000, p. 1912; 1999, p. 153). Bankruptcy cost  $B(V, V_B)$  is similar to that in Leland and Toft (1996). Equity value  $E(V, V_B, T)$  is given by

$$E(V, V_B, T) = \left(V + \left(1 - \frac{(1 - \tau_C)(1 - \tau_E)}{1 - \tau}\right) \frac{C}{r} \left[1 - \left(\frac{V_B}{V}\right)^{a+z}\right] - \beta V_B \left(\frac{V_B}{V}\right)^{a+z} - D(V, V_B, T)\right) \cdot \left(1 - \tau_{EC} \left(\frac{V_B}{V}\right)^{a+z}\right)^{-1}. \quad (12)$$

Appendix A provides the derivation of  $h(V, V_B)$  and  $E(V, V_B, T)$ ; all appendices are available online at <http://mansci.pubs.informs.org/ecompanion.html>.

Similar to Leland (1994), the model allows the firm to operate with negative net worth as long as there is enough asset value or new capital raised from equity holders to meet the debt interest payment before maturity. This setup differs from those models with a positive net worth covenant (e.g., Merton 1974, Longstaff and Schwartz 1995). In this endogenous default model, equity holders optimally decide on the timing of default. The default boundary  $V_B$  can be solved by using the smooth-pasting condition

$$\frac{\partial E(V, V_B, T)}{\partial V} \Big|_{V=V_B} = 0. \quad (13)$$

Without loss of generality, we set the initial unlevered firm value  $V$  equal to 100. We then impose the following par-bond constraint that new debt is sold at par value,

$$d(V, V_B, c, p, T)|_{V=100} = p(T). \quad (14)$$

Note that  $c$  and  $p$  are functions of  $V$ ,  $V_B$ , and  $T$ . Using (12) and applying the smooth-pasting condition in (13), we have

$$\begin{aligned} \frac{\partial E}{\partial V} \Big|_{V=V_B} &= \tau_{EC}(a+z) \frac{D(V, V_B, T) - (1 - \beta)V_B}{(1 - \tau_{EC})^2} \\ &+ \left(1 + \left[1 - \frac{(1 - \tau_C)(1 - \tau_E)}{1 - \tau}\right] \frac{C(a+z)}{rV_B} + \beta(a+z) - \int_0^T \Omega dt\right) \cdot (1 - \tau_{EC})^{-1} = 0, \end{aligned} \quad (15)$$

where

$$\Omega = \frac{\partial d(V, V_B, c, p, T)}{\partial V} \Big|_{V=V_B}. \quad (16)$$

The expression of  $\Omega$  is given in Appendix B. Given  $\Omega$ ,  $T$ , and setting the initial asset value  $V|_{t=0} = 100$ , we are left with three unknowns: default trigger  $V_B$ , coupon flow  $c$ , and the face value of newly issued debt  $p$ . The par-bond condition in (14) allows us to find the solution for  $p$ :

$$\begin{aligned} p(T) &= \left(\frac{(1 - \tau)c}{r} - e^{-rt} \frac{(1 - \tau)c}{r} [1 - F(T)]\right) \\ &+ \left[(1 - \alpha\tau)\rho(T)V_B - \frac{(1 - \tau)c}{r}\right] G(T) \\ &\cdot (1 - \alpha\tau\{e^{-rt}[1 - F(T) + G(T)]\} - e^{-rt}(1 - \alpha\tau)[1 - F(T)])^{-1}. \end{aligned} \quad (17)$$

Because  $c$  is a function of  $V_B$ , this leaves  $V_B$  as the only unknown variable. The optimal value of  $V_B$  can be found by maximizing the firm value  $W$ :

$$\frac{\partial W(V, V_B, T)}{\partial V_B} \Big|_{V=100} = 0. \quad (18)$$

The firm value is a strictly concave function of default trigger  $V_B$ . Once the default boundary  $V_B$  is determined, we can solve for the optimal values of debt and equity, and hence the optimal leverage ratio  $l = D/W$ .<sup>5</sup> In general,  $V_B$  depends on debt maturity, personal and corporate income taxes, risk-free rate, and bankruptcy costs. The personal tax rates are embedded in  $D$  (and  $p$ ), which ultimately affects the default boundary. Through their effect on  $V_B$ , personal income

<sup>5</sup> The smooth-pasting condition ensures that equity value is also maximized when firm value is maximized.

taxes affect firm value  $W$  and other characteristics. Finally, the yield spread for the bond issue with maturity  $T$  is calculated as  $YS = c/d(V, V_B, T) - r_f$ , where  $r_f$  is the before-tax interest rate of riskless (Treasury) debt with the same maturity.

### 3. Numerical Analysis

Because the model does not have a closed-form solution, we conduct numerical analysis to examine personal tax effects on corporate capital structure and bond yields. In each simulation, we partition each year into 100 equal discrete intervals  $\Delta t$ . In solving some equations, we employ the bisection method to obtain the solution.<sup>6</sup> We first demonstrate how personal taxes affect the optimal capital structure, the firm's value, debt price, and yield spreads under different scenarios of interest rates, payout ratios, personal tax rates, and bankruptcy recovery rates. This exercise allows us to compare our findings directly with those of Leland and Toft (1996), who assume away personal taxes and carry no model calibration. Later, we calibrate the model to be consistent with historical data and use it to assess the effects of taxes on corporate bond yield spreads.

Capital gains and interest income from corporate bonds are subject to both state and federal taxes, whereas Treasury bond returns are subject only to federal taxes. In reality, investors are subject to different tax rates, depending on their income status.<sup>7</sup> Returns associated with pension funds, IRA accounts, and insurance annuities are tax deferred. Also, institutions and firms are typically not permitted to hold more than a small fraction of high-risk bonds. In fact, some institutions are barred from investing in corporate bonds. These constraints may limit the participation of institutional investors in corporate bonds. Thus, the marginal tax rate may vary for different types of investors and for corporate bonds with different ratings.<sup>8</sup> In the simulation, we first set the personal income tax rates to a plausible range of 0%–35% to cover a variety of potential clienteles. The capital gains tax rate is a fraction of the income tax rate  $\alpha\tau$ ,

where  $\alpha$  is set equal to 0.5. Later, we use the empirical tax rates documented by Graham (1999, 2000) to assess tax effects. In the following, we investigate personal tax effects on the firm's optimal financing decisions, firm characteristics, and bond yields.

#### 3.1. Effects of Personal Taxes on Optimal Leverage and Other Firm Characteristics

An important issue in corporate finance is whether personal taxes affect the firm's leverage when there is default risk. In the absence of taxes and default, Modigliani and Miller (1958; hereafter MM) show that the firm value is independent of capital structure. With only corporate income taxes, the firm value increases as the leverage increases (see MM 1958, 1963). Miller (1977) extends the MM (1958) model to include personal and corporate taxes, and concludes that capital structure decisions are irrelevant. Specifically, in equilibrium, the corporate tax savings from debt interest are completely offset by the tax penalty on investors holding debt instead of equity. Thus, changes in capital structure have no effect on the firm value. These models provide qualitative guidance but ignore default risk and bankruptcy costs. Considering the effects of the endogenous default strategy and bankruptcy costs may alter the inference regarding the personal tax effect. Recently, Cooper and Davydenko (2003) found that taxes and default risk are independent under the assumption that leverage is not affected by personal taxes, and income and capital gains tax rates are the same. It can be shown that this conclusion will not hold if either of their assumptions is violated (see Appendix C). In particular, if there is an interactive relationship between personal taxes and leverage, personal taxes will affect leverage, default risk, and credit spreads jointly.

We begin with the analysis of personal tax effects on the firm's financing decisions and characteristics. We set asset volatility  $\sigma = 0.2$ , interest rate  $r = 7.5\%$ , and payout ratio  $\delta = 7\%$ . These parameter values are similar to those used by Leland and Toft (1996). In addition, corporate income tax rate  $\tau_C = 35\%$ , personal income tax rate for debt  $\tau$  ranges from 0%–35%, and effective equity income tax rate  $\tau_E = (1 - \delta)\alpha\tau + \delta\tau$ . We choose bankruptcy costs  $\beta = 20\%$  based on the findings from Andrade and Kaplan (1998). Table 1 reports the results for the effects of taxes on firm characteristics under different personal tax rates displayed in column 1 and varying debt maturities (Panels A–D).

For ease of illustration, Figure 1 plots the firm value as a function of leverage with different personal tax rates and maturities (0.5, 5, 10, and 20 years). As shown, personal taxes affect the firm value and the optimal capital structure. The optimal leverage ratio (corresponding to the maximum firm value) decreases with the personal tax rate. For example, for maturity

<sup>6</sup> The bisection method is very efficient for finding the solution of nonlinear equations (see Judd 1998, p. 147). It is an iterative method, which always converges, given the right range for the domain of the independent variable.

<sup>7</sup> Under the current U.S. tax laws, the highest corporate and individual income tax rates are both equal to 35%. For most states, maximum marginal state income tax rates range between 5% and 10%.

<sup>8</sup> Because of regulation, some institutions are prohibited from investing in corporate bonds, and so they may just invest in Treasury securities. It happens that these institutional investors are often tax exempt (e.g., pension funds) or are in the lower tax bracket. Thus, marginal investors in corporate bonds may have higher tax rates than those in Treasury bonds. The authors thank an anonymous referee for pointing out this fact.



**Table 1** Endogenously Determined Firm Characteristics Under Different Personal Taxes

(1)	(2)	(3)	(4)	(5)	(6)	(7)
Personal income tax rate (%)	Firm value $W$	Total debt $D$	Total annual coupon $C = cT$	Total principal $P = pT$	Optimal bankruptcy boundary $V_b$	Optimal leverage ratio (%)
Panel A: Firm issues debt with maturity $T = 6$ months						
0	107.624	42.504	2.656	35.420	40.227	39.493
5	107.325	42.861	2.700	35.717	39.762	39.936
10	107.001	42.733	2.715	35.611	38.988	39.937
15	106.666	42.091	2.700	35.076	37.905	39.461
20	106.343	40.925	2.653	34.104	36.517	38.484
25	106.054	39.265	2.576	32.721	34.849	37.023
30	105.827	37.165	2.471	30.971	32.937	35.118
35	105.695	34.704	2.343	28.920	30.824	32.834
Panel B: Firm issues debt with maturity $T = 5$ years						
0	111.19	57.99	4.76	56.47	50.89	52.15
5	110.16	54.82	4.46	53.41	48.64	49.76
10	109.21	51.49	4.16	50.19	46.22	47.15
15	108.36	48.05	3.86	46.87	43.68	44.34
20	107.62	44.54	3.57	43.49	41.04	41.38
25	107.02	41.05	3.29	40.11	38.38	38.36
30	106.56	37.64	3.03	36.81	35.75	35.33
35	106.26	34.37	2.78	33.64	33.18	32.35
Panel C: Firm issues debt with maturity $T = 10$ years						
0	113.24	61.68	5.33	60.54	50.27	54.47
5	111.88	58.21	5.01	57.11	48.28	52.03
10	110.64	54.71	4.69	53.66	46.19	49.45
15	109.53	51.17	4.38	50.18	43.99	46.72
20	108.55	47.58	4.06	46.66	41.66	43.83
25	107.73	43.95	3.75	43.11	39.21	40.80
30	107.07	40.30	3.45	39.55	36.65	37.64
35	106.61	36.67	3.14	36.01	34.01	34.40
Panel D: Firm issues debt with maturity $T = 20$ years						
0	114.76	63.76	5.60	63.09	48.73	55.56
5	113.17	59.98	5.27	59.28	46.90	53.00
10	111.71	56.23	4.94	55.51	44.98	50.34
15	110.41	52.51	4.61	51.78	42.96	47.56
20	109.26	48.79	4.29	48.07	40.82	44.65
25	108.28	45.08	3.98	44.39	38.58	41.63
30	107.49	41.38	3.67	40.72	36.23	38.50
35	106.89	37.69	3.36	37.08	33.77	35.26

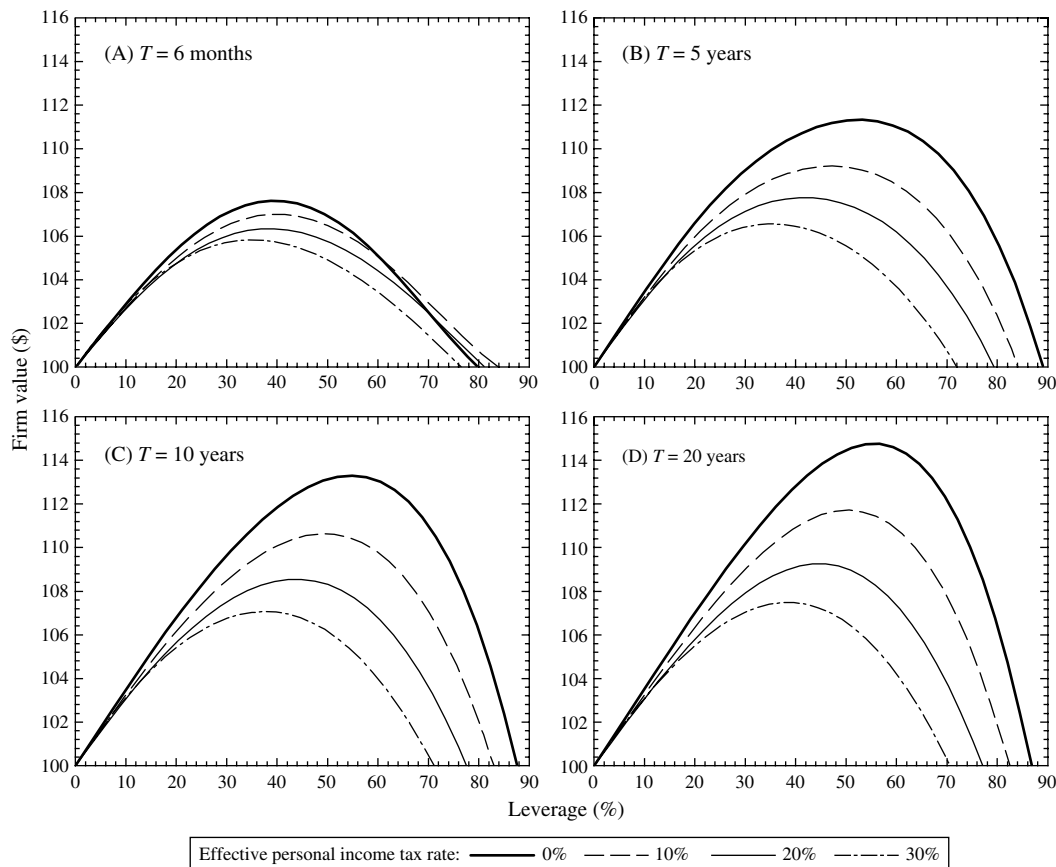
*Note.* This table reports the optimal firm characteristics as a function of personal income tax rates  $\tau$  displayed in column 1 for firms issuing debt with maturity  $T$  equal to 6 months (Panel A), 5 years (Panel B), 10 years (Panel C), and 20 years (Panel D). We set before-tax risk-free interest rate  $r = 7.5\%$ , payout ratio  $\delta = 7\%$ , asset volatility  $\sigma = 20\%$ , bankruptcy cost ratio  $\beta = 20\%$ , and corporate tax rate  $\tau_c = 35\%$ . The initial value of  $V$  is set to 100. Personal income tax rates for debts  $\tau$  and effective equity tax rates  $\tau_E$  are linked by the formula  $\tau_E = (1 - \delta)\alpha\tau + \delta\tau$  (see Engel et al. 1999, Graham 2003).

$T = 6$  months (Panel A), an increase in the ordinary income tax rate  $\tau$  from 0% to 35% causes the optimal leverage to drop from 40% to 33%. For maturity  $T = 20$  years, it reduces the optimal leverage from 56% to 35%. Results show that the impact of taxes increases with maturity and the firm value decreases with the personal income tax rate.

The reasons for these outcomes are as follows. An increase in personal tax rate  $\tau$  raises the cost of debt because firms must issue debt at a lower price to com-

pensate investors for the higher tax burden. The drop in debt price reduces the benefit of the corporate tax shield associated with leverage. This negative effect on the tax shield increases with maturity.<sup>9</sup> As a result, an increase in  $\tau$  has a larger impact on the leverage of the firm issuing long-term debt. On the other hand, a higher  $\tau$  raises the effective tax rate on equity  $\tau_E$ ,

<sup>9</sup> This is because long-maturity bonds carry more coupons and a larger tax shield.

**Figure 1** Firm Value as a Function of Leverage with Different Personal Tax Rates

*Note.* The value of the firm is plotted as a function of leverage for firms issuing debt with varying maturity  $T$ . For each maturity  $T$ , four personal tax rates are applied. The parameter choices are similar to those in Leland and Toft (1996): the before-tax risk-free interest rate  $r = 7.5\%$ , payout ratio  $\delta = 7\%$ , asset volatility  $\sigma = 20\%$ , and corporate income tax rate  $\tau_c = 35\%$ . We choose the bankruptcy cost ratio  $\beta = 20\%$  based on Andrade and Kaplan (1998). The initial value of  $V$  is set to 100, and the default trigger  $V_B$  is determined endogenously. The optimal leverage corresponds to the maximum firm value. The effective equity tax rate is a weighted average of ordinary income and capital gains tax rates, where the weight is the payout ratio, i.e.,  $\tau_E = (1 - \delta)\alpha\tau + \delta\tau$ .

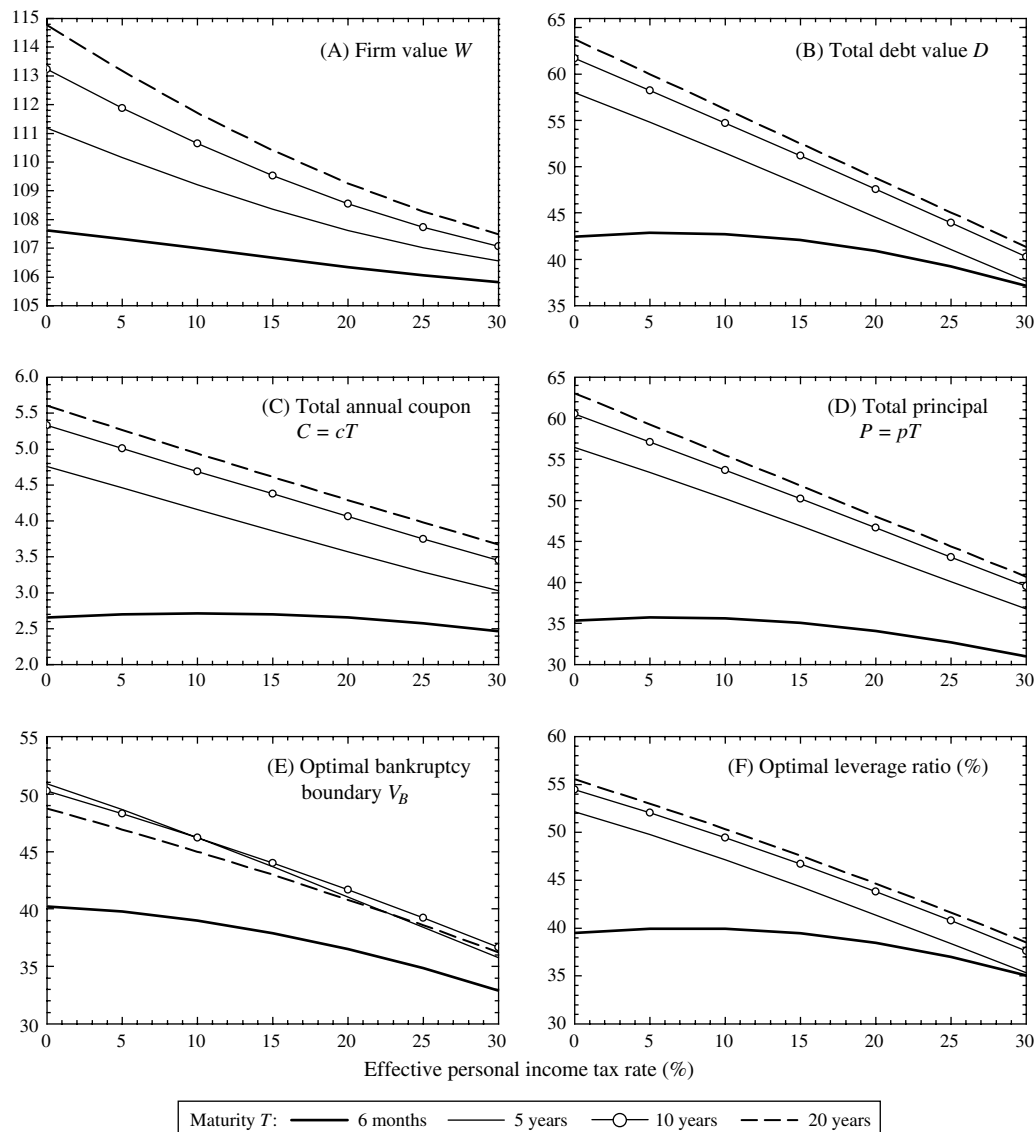
which partially offsets the negative effect of  $\tau$  on debt financing. However, because the tax rate on debt is higher than that on equity, an increase in tax rate  $\tau$  still makes it less attractive for the firm to use debt.

Changes in capital structure are linked to changes in other firm characteristics. Columns (2)–(6) of Table 1 report changes in firm characteristics because of changes in tax rates. For ease of illustration, we also plot them in Figure 2. Panel F in Figure 2 shows that the optimal leverage level is positively related to debt maturity and negatively related to personal tax rates. When  $\tau$  increases, both the optimal leverage (Panel F) and default boundary  $V_B$  (Panel E) decrease because the cost of debt relative to equity increases. Although lower leverage (and lower  $V_B$ ) reduces the risk of debt, there is a decline in debt value  $D$ , principal value  $P = pT$ , and total annual coupon liability  $C = cT$  when the tax rate increases, as portrayed in Panels B, C, and D, respectively. Results suggest that the negative effect of the tax penalty on debt cost outweighs the positive effect of lower default

risk. Similarly, the firm value  $W$  decreases with  $\tau$  because of the decrease in leverage and the reduction in the tax shield. Firms issuing debts with longer maturity experience a bigger drop in these values. In general, these effects associated with taxes increase at a decreasing rate as maturity rises.

A closer examination of the optimal default boundary  $V_B$  in Figure 2 (Panel E), the total principal  $P$  (Panel D), and the total outstanding debt value  $D$  (Panel B) shows that  $V_B$  can be set either below  $P$  (for  $T = 10$  and 20 years) or above it (for  $T = 6$  months). Default is triggered not because the firm value  $W$  falls beneath  $P$ , but because the anticipated equity appreciation does not warrant the additional contribution required from equity holders to avoid default on debt service payments. The anticipated equity appreciation is lower over a shorter time horizon. Consequently, a firm issuing short-term debt has to set a higher default trigger  $V_B$  to raise debt capital. Default may occur despite the fact that net worth is positive. Conversely, a firm issuing long-term debt can set  $V_B$

Figure 2 Firm Characteristics as a Function of Personal Income Tax Rate



*Note.* Firm characteristics are plotted as a function of personal income tax rate for firms issuing debt with varying maturity  $T$ . We set the before-tax risk-free interest rate  $r = 7.5\%$ , payout ratio  $\delta = 7\%$ , asset volatility  $\sigma = 20\%$ , bankruptcy cost ratio  $\beta = 20\%$ , and corporate income tax rate  $\tau_c = 35\%$ . The initial value of  $V$  is set to 100, and the default trigger  $V_B$  is determined endogenously. The effective equity tax rate is a weighted average of ordinary income and capital gains tax rates, where the weight is the payout ratio, i.e.,  $\tau_E = (1 - \delta)\alpha\tau + \delta\tau$ .

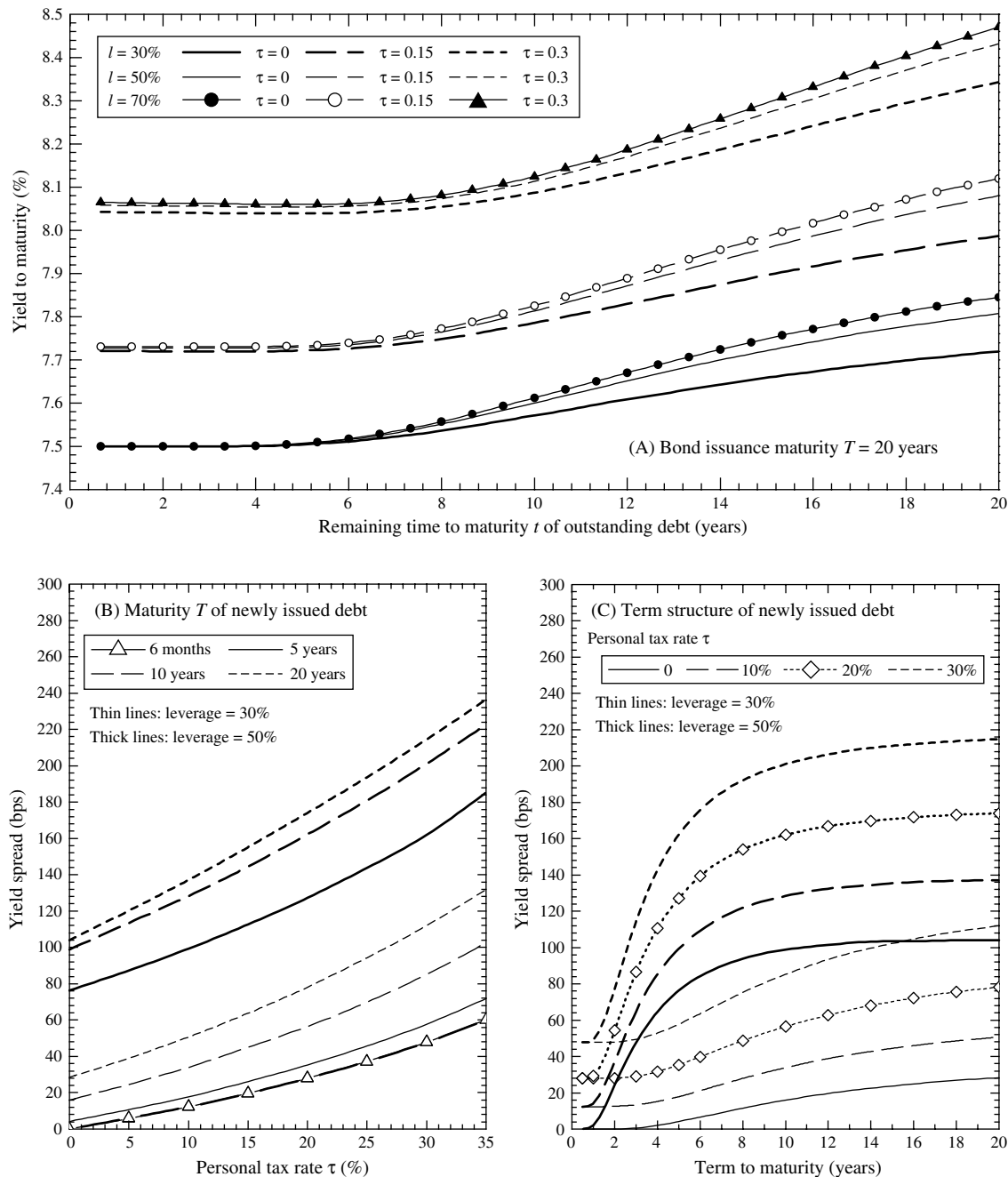
lower than  $P$  because higher long-term equity appreciation makes it easier to meet its debt obligations.

### 3.2. Yield on Outstanding Debt and Term Structure of Spreads

We next examine yields of both outstanding and newly issued debts. The upper panel of Figure 3 plots yields to maturity for outstanding debts (with the issuance maturity  $T = 20$  years) as a function of maturity for firms at different levels of leverage and under three tax rate scenarios: 0%, 15%, and 30%. Bond yield increases with time to maturity and leverage. Given a tax rate, the marginal impact of leverage on yields increases as leverage rises. When  $\tau = 0$ , we

obtain similar results to Leland and Toft (1996). In the absence of personal taxes and accounting information noise, yields to maturity on debts issued by the firms with different leverage levels merge as debts approach maturity because default risk becomes negligible for all debts. As time to maturity rises, the yield spread widens because of increasing differences in default risk. When there are personal taxes, yield increases with  $\tau$ . The marginal impact of the tax rate on yields is larger as time to maturity increases.

The lower panels of Figure 3 show yield spreads on newly issued debts with different maturities and tax rates for firms with the leverage of 30% and 50%. In the lower left panel, the yield spread curve is

**Figure 3** Yield to Maturity on Outstanding Debt and Yield Spread on Newly Issued Debt

*Note.* The upper panel shows the yield to maturity on a 20-year bond under three leverage ratios ( $l = 30\%$ ,  $50\%$ , and  $70\%$ ) and three personal tax rates ( $\tau = 0\%$ ,  $15\%$ , and  $30\%$ ). The lower panels are for newly issued debt for firms with leverage ratio of  $30\%$  and  $50\%$ . The lower left panel plots the spread for four issuance maturities ( $T = 0.5, 5, 10$ , and  $20$  years) as a function of personal tax rate. The lower right panel plots the term structure of spreads under four different personal tax rates ( $\tau = 0\%$ ,  $10\%$ ,  $20\%$ , and  $30\%$ ). The default boundary  $V_B$  is endogenously determined. Before-tax risk-free interest rate  $r = 7.5\%$ , payout ratio  $\delta = 7\%$ , asset volatility  $\sigma = 20\%$ , bankruptcy cost ratio  $\beta = 20\%$ , and corporate income tax rate  $\tau_C = 35\%$ . The initial value of  $V$  is set to 100. The effective equity tax rate is a weighted average of ordinary income and capital gains tax rates, where the weight is the payout ratio, i.e.,  $\tau_E = (1 - \delta)\alpha\tau + \delta\tau$ .

drawn as a function of the personal tax rate  $\tau$  for four issuance maturities:  $T = 6$  months, 5 years, 10 years, and 20 years. The results show that spreads increase with the personal tax rate and leverage. Note that when maturity is six months, the spread curve for the leverage of  $50\%$  is only slightly higher than that for

the leverage of  $30\%$  and so it is almost indistinguishable. In the lower right panel, the spread is plotted as a function of issuance maturity  $T$  under different personal tax rates. The spread curve has a different shape for different leverage ratios. At a fixed personal tax rate, the spread in basis points increases with issuance

maturity, but the rate of increase depends on the leverage ratio and maturity. Moreover, the marginal impact of the tax rate on spreads is higher for the high-leverage firm.

We next examine the sensitivity of yield spread to the risk-free interest rate and payout ratio. In addition to the baseline case (see the lower panels of Figure 3) with  $r = 7.5\%$  and payout  $\delta = 7\%$ , we examine four other scenarios: low interest rate and payout ratio ( $r = 4.5\%$  and  $\delta = 4\%$ , Panel A), high interest rate and payout ratio ( $r = 9\%$  and  $\delta = 7\%$ , Panel D), and medium interest rate with low and high payouts, respectively ( $r = 6\%$  and  $\delta = 4\%$ , Panel B, and  $r = 6\%$  and  $\delta = 5.5\%$ , Panel C). Personal tax rates are set to 0%, 10%, 20%, and 30%. Figure 4 plots the results.

Comparing the results in Panels A and B of Figure 4 (where  $\delta$  is fixed at 4%) shows that a higher  $r$  generally results in a lower yield spread, except for short-maturity bonds. A higher  $r$  implies that the firm value is likely to drift to a higher level in the future. Because bond investors expect that the firm will have less difficulty to service the debt, they require a lower premium. The effect of interest rate on spread increases with maturity because the firm value will rise to a higher level over a longer horizon.

Comparing Panel B to C of Figure 4, we find that the spread increases with the payout ratio. The spread

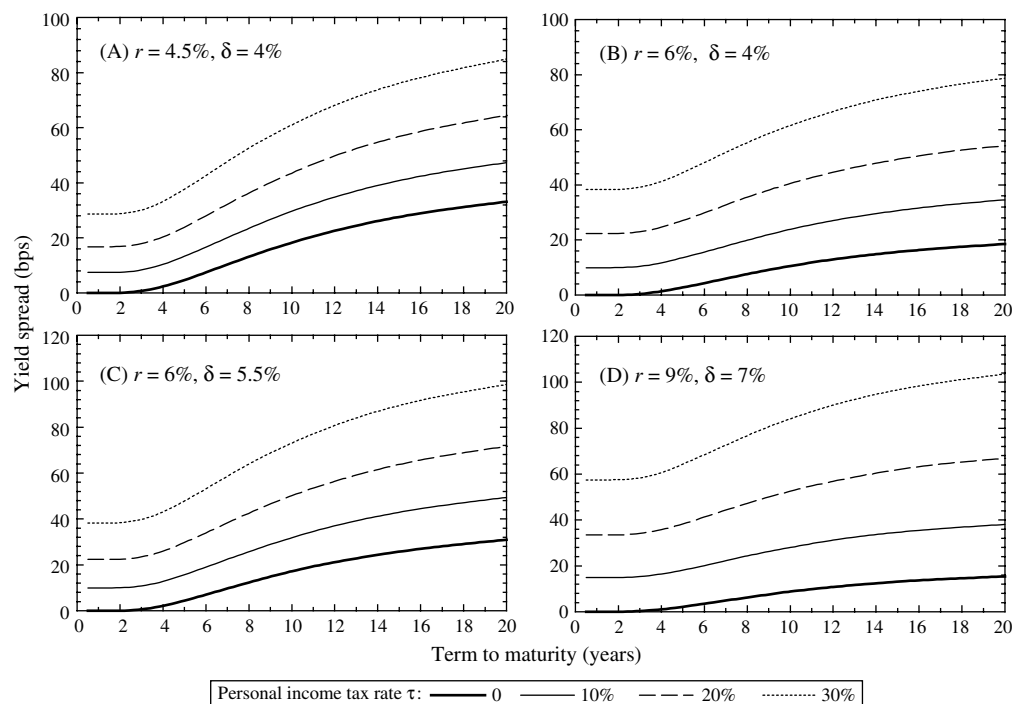
increases because the reduced net drift rate  $r - \delta$  implies that the firm value  $W$  cannot reach the same level as it does with the original drift rate. Because the firm is expected to have greater difficulty in raising equity capital for its debt payments, investors require a higher yield on the debt. This effect is greater for bonds with longer issuance maturity.

In controlling for the level of the net drift rate, the spread increases when both  $r$  and  $\delta$  increase. With the same net drift rate  $r - \delta = 2\%$ , the spread increases from Panel B to D. Because the net drift rate does not change, the growth rate of the firm value is not affected. However, an increase in the interest rate raises the discount rate for coupon payments, causing the debt value to decrease and yield the spread to widen. This impact increases with maturity and the personal tax rate.

#### 4. Model Calibration and Tax Effects on Spreads

We have shown that personal taxes can markedly affect the default boundary, the firm's characteristics, and yield spreads. These results provide relevant information for direct comparison with the Leland-Toft (1996) model and other term structure models that ignore personal taxes. However, when

Figure 4 Yield Spread on Newly Issued Debt as a Function of Personal Income Tax Rate Under Alternative Interest Rates and Payout Ratios



*Note.* This figure shows the term structure of yield spreads under different tax rates for firms with a leverage ratio of 30%. Four panels represent low interest rate and low payout (Panel A), high interest rate and high payout (Panel D), medium interest rate with low payout (Panel B), and medium interest rate with relatively high payout (Panel C), respectively. Asset volatility  $\sigma = 20\%$ , bankruptcy cost ratio  $\beta = 20\%$ , corporate income tax rate  $\tau_c = 35\%$ , and personal income tax rates are from 0% to 30%. The effective equity tax rate and personal income tax rate  $\tau$  are linked by  $\tau_E = (1 - \delta)\alpha\tau + \delta\tau$ . The initial value of  $V$  is set to 100, and the default trigger  $V_b$  is determined endogenously.



using the model to predict yield spread, it is important to verify whether the model-implied default probability and the loss rate given default are indeed consistent with historical default loss experience. Huang and Huang (2003) provide an excellent example in this respect by showing that proper model calibration is essential to generate robust results and consistent estimates of credit yield spreads. In this section, we adopt a similar approach to calibrate our model.

#### 4.1. Model Calibration

We calibrate our model to ensure that it will predict an expected level of default loss equal to that actually experienced by bondholders. We employ the historical default probability and average default loss rate for each rating class as target values for model calibration. The model is required to generate default parameters that match these actual data associated with different rating classes. In addition, we make use of the historical equity premium associated with each rating class. In our model, the default probability depends on the firm's asset risk premium, which is unobservable, but closely linked to the equity premium. Because equity market data are more reliable than corporate bond market data, the equity premium data provide important information to estimate the firm's asset risk premium more accurately. A deviation of our calibration from Huang and Huang's (2003) procedure is that we let the model endogenize the firm's leverage to generate the optimal capital structure consistent with historical default experience. This procedure grants the model maximal flexibility in fitting default data. The method of calibration is explained in more detail in Appendix D. The target values of the selected parameters and average historical spreads for each rated bond with maturity of 10 years are displayed in Table 2. We focus on 10-year bonds because the ratings provided by Moody's are based on 10-year default frequencies. All the data except recovery rates are acquired from Huang and Huang (2003). We employ a bankruptcy cost ratio ( $\beta$ ) equal to 20% of the firm value as suggested by Andrade and Kaplan (1998). Finally, as noted by Huang and Huang (2003), average historical corporate spreads are not precise, but they serve as important references for comparison with the yield spreads calculated by the model.

#### 4.2. Effects of Personal Taxes on Spreads

We initially calibrate the model by assigning an exogenous tax rate. Table 3 reports the results for the debt with issuance maturity  $T = 10$  years, interest rate  $r = 7.5\%$ , and payout ratio  $\delta = 7\%$ . These results are calculated under four personal tax scenarios: no taxes, taxes on debt but not on equity, taxes on both debt and equity, and tax rates based on Graham's (1999) empirical estimates.

**Table 2** Parameters for Model Calibration

Credit rating	Target parameters			Average yield spread (bps)
	Recovery rate (%)	Equity premium (%)	Cumulative default probability (%)	
AAA	80	5.38	0.77	63
AA	80	5.60	0.99	91
A	80	5.99	1.55	123
BBB	80	6.55	4.39	194
BB	80	7.30	20.63	320
B	80	8.76	43.91	470

*Note.* This table shows the target values of the parameters that our model will conform to after calibration. The last column includes the average observed yield spreads. Except for the recovery rates, the data presented here are directly taken from Huang and Huang (2003) for the period 1973–1993. Andrade and Kaplan (1998) and Eom et al. (2003) indicate that the cost of financial distress  $\beta$  is in the range of 15% to 20% of the firm's going concern value. We choose 80% as the target recovery rate, which is close to that used by Huang and Huang (2003) in the Leland-Toft (1996) model for A and Baa bonds. Note that the recovery rate here is measured as the percentage of the firm value at default.

Column 3 of Table 3 shows the yield spreads predicted by the model calibrated without personal taxes ( $\tau = 0$ ). Because personal taxes are assumed away, these model-inferred spreads largely reflect default premiums. The results show that default risk explains about 31%–67% of the observed spreads for bonds of different ratings. The proportion of spread explained by the model tends to be higher for junk bonds. This finding is consistent with Huang and Huang (2003).

Column 4 of Table 3 reports the estimated spread where we set  $\tau = 20\%$  and  $\tau_E = 0$ .<sup>10</sup> Here, we assume no taxes on equity. Adding personal taxes for bond returns generally increases the predicted spread. The increase is larger for investment-grade bonds. For example, the predicted spread for AAA bonds increases from 26 bps to 42 bps, a 62% increase. The proportion of observed spread explained by the model ranges from 36% to 67%.

The results in column 4 of Table 3 assume no taxes on equity returns, which exaggerates the tax disadvantage of corporate bond investments. In reality, investors' equity returns are also subject to taxes. We next incorporate taxes for equity returns (column 5). An ordinary income tax rate ( $\tau$ ) of 20% and payout ratio ( $\delta$ ) of 7% would imply an effective equity return tax rate  $\tau_E$  of 10.7% according to the formula  $\tau_E = (1 - \delta)\alpha\tau + \delta\tau$  (see Graham 2003).<sup>11</sup> Results in

<sup>10</sup> This tax rate is within the range of estimates by Liu et al. (2004). The purpose here is to set a reasonable tax rate to see its impact on yield spread. Later, we use Graham's (1999) empirical tax rate, which will be slightly higher.

<sup>11</sup> This formula implicitly assumes that there is a certain marginal investor who owns both equity and debt.

**Table 3** Yield Spreads, Default, and Tax Premiums

(1)	(2)	(3)	(4)	(5)	(6)
Rating	Observed spread (bps)	Default only (bps), (%)	$\tau = 20\%$ ; $\tau_E = 0$ (bps), (%)	$\tau = 20\%$ ; $\tau_E = 10.7\%$ (bps), (%)	$\tau = 22.43\%$ ; $\tau_E = 12\%$ (bps), (%)
AAA	63	26 (41%)	42 (67%)	51 (81%)	55 (87%)
AA	91	30 (33%)	45 (49%)	55 (60%)	59 (65%)
A	123	38 (31%)	50 (41%)	63 (51%)	67 (54%)
BBB	194	64 (32%)	69 (36%)	88 (45%)	92 (47%)
BB	320	160 (50%)	152 (48%)	176 (55%)	180 (56%)
B	470	315 (67%)	317 (67%)	330 (71%)	334 (71%)

*Note.* The bankruptcy cost ratio  $\beta = 20\%$ , interest rate  $r = 7.5\%$ , and payout ratio  $\delta = 7\%$ . Column 2 shows the average observed spreads on 10-year bonds and column 3 reports the spreads generated by the model calibrated without taxes. Columns 4–6 report the spreads generated by the model for different personal tax scenarios. In column 4, equity tax is set equal to zero. In column 5, equity tax rate is estimated as a weighted average of personal income tax and capital gains tax rates, i.e.,  $\tau_E = (1 - \delta)\alpha\tau + \delta\tau$ . In column 6, we adopt the average equity tax rate  $\tau_E = 12\%$  estimated by Graham (1999). The numbers in parentheses express the model-generated spread as a percentage of the observed spread.

column 5 show that personal taxes explain an even greater proportion of the observed spread after incorporating this equity tax rate. This is because taxes on equity increase the firm's incentive to issue more debt, thereby offsetting the negative effect of debt taxes on leverage. For example, compared to the case without taxes, the predicted spread for AAA bonds increases from 26 bps to 51 bps, a 96% increase. The proportion of observed spreads explained by the model now ranges from 45% to 81%.

Goldstein et al. (2001) also consider taxes on equity in their study. Our model differs from theirs in several aspects. First, we include state taxes in the personal tax package for bond investors. Second, we allow for tax rebates when default occurs. Third, our model permits an interactive effect between default and taxes. The interactive effect arises mainly from the differential treatments of ordinary income and capital gains. We show in Appendix C that this interactive effect has a positive impact on yield spread. Because these factors are left out, the yield spread between corporate and Treasury bonds mainly reflects the tax-induced default premium in the Goldstein et al. (2001) model.<sup>12</sup> An increase in tax rate would reduce the firm's leverage and lower default, resulting in a lower yield spread. In addition, in their model, the tax benefit of debt becomes smaller as cash flow falls to a certain level and the firm loses part of its tax shelter, which leads to an optimally lower leverage and lower yield spreads (see also Strebulaev 2004). By contrast, our model captures the effects of the three important tax factors. Including state taxes leads to an increase in corporate bond yield as investors require a compensation for the increased tax burden. If the com-

bined effect of state taxes and the tax-default interaction (including tax rebates at default) outweighs the effect of reduced leverage, yield spread will increase. Our results show that in most cases, corporate bond yield spread increases when both debt and equity taxes are taken into account. The only exception is for BB bonds in column 4 of Table 3, where the leverage effect dominates, causing a decrease in spread.

The results above are obtained by assigning an arbitrary income tax rate. Graham (1999) estimates that the marginal equity return tax rate  $\tau_E$  is close to 12%. This implies an ordinary income tax rate  $\tau$  of 22.43% given that  $\tau_E = (1 - \delta)\alpha\tau + \delta\tau$  ( $\delta$  is 7% and  $\alpha$  is 0.5). Using these empirical tax rates, we re-estimate spreads, which are displayed in column 6 of Table 3. The spreads predicted by the model increase further, ranging from 47% to 87%. Using more realistic debt and equity income tax rates raises the size of predictive spreads considerably, especially for investment-grade bonds. For example, compared to the model without personal taxes, the proportion of observed spreads explained by the model with taxes increases from 41% to 87% for AAA bonds, 33% to 65% for AA bonds, 31% to 54% for A bonds, and 32% to 47% for BBB bonds. On the other hand, taxes add only about 6% to the predicted spreads for junk bonds. This is because junk bonds have high default risk where the default premium accounts for a large portion of the spread. Given the large default premium, the proportion of the added tax premium to the total spread is relatively small.

The results in column 6 of Table 3 show that there is still some spread left unexplained, particularly for medium- and low-grade bonds. Part of the remaining spread could be attributed to liquidity risk unaccounted for by the model. Recently, de Jong and Driessen (2004) reported estimates of liquidity premiums for bonds of different ratings ranging from about

<sup>12</sup> This is because both Treasury and corporate bonds are subject to the federal tax rate because state taxes are ignored.

**Table 4** Personal Income Tax Rates Implied by the Observed Spreads on 10-Year Bonds

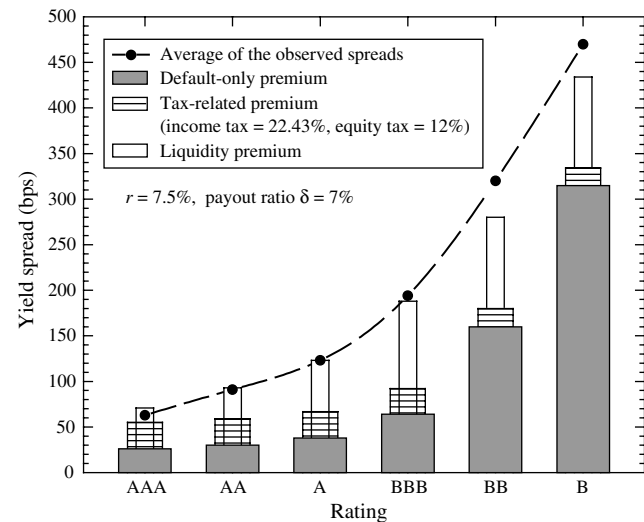
(1) Credit rating	(2) Observed spread (bps)	(3) Liquidity component (bps)	(4) Nonliquidity component (bps)	Implied personal tax rates	
				(5) Income tax rate (%)	(6) Equity tax rate (%)
AAA	63	16	47	17.4	9.3
AA	91	34	57	21.1	11.3
A	123	56	67	22.5	12.0
BBB	194	96	98	26.4	14.1
BB	320	100	220	40.7	21.8
B	470	100	370	35.8	19.2
Average implied tax rates		All ratings		27.3	14.6
		Investment grade		21.9	11.7

*Note.* This table reports the implied income tax rates obtained by calibrating the model to match the nonliquidity spread component. The equity tax rate  $\tau_E = (1 - \delta)\alpha\tau + \delta\tau$ , bankruptcy cost ratio  $\beta = 20\%$ , interest rate  $r = 7.5\%$ , and payout ratio  $\delta = 7\%$ . The liquidity component of spread in column 3 is taken from de Jong and Driessen (2004). For AAA and AA bonds, because of a large discrepancy between intermediate-term (5 years) and long-term bonds (10–22 years), we take the average of them. The nonliquidity component in column 4 is obtained by subtracting the liquidity spread in column 3 from the observed spread in column 2.

16 bps to 100 bps for bonds with ratings of AAA to B (see column 3 of Table 4). If we add these liquidity premium estimates to the spreads predicted by the model in column 6, we can almost perfectly predict the spreads for all investment-grade bonds. Even for junk bonds, the estimated spreads are on average slightly more than 90% of the observed spreads. Figure 5 plots the sum of the default, tax, and liquidity spreads for bonds in each rating class.

Furthermore, we can calibrate the model to generate spread estimates consistent with historical spreads by allowing tax rates to be endogenously determined. This procedure contrasts with that in Table 3 where tax rates are exogenously assigned. We can then compare the model-implied tax rates with empirical tax rates and evaluate the model's performance. If the implied tax rates are indeed close to the empirical tax rates (e.g., Graham 1999), it would suggest that the model performs well. However, to obtain more reasonable implied tax rates, we must account for the potential liquidity effect to avoid its being misconstrued as the tax effect. To control for the liquidity effect, we adjust the observed spread by the liquidity premium estimated by de Jong and Driessen (2004) to come up with the nonliquidity component of spread, which is reported in column 4 of Table 4. We then calibrate the model against this adjusted spread to obtain the implied marginal income tax rate.

Column 5 of Table 4 reports the implied tax rates for bonds of different ratings. The implied ordinary

**Figure 5** Observed Spreads and Model-Generated Spreads After Calibration

*Note.* The bankruptcy cost ratio  $\beta = 20\%$ , and the corporate income tax rate  $\tau_C = 35\%$ . For each bond rating, we plot the model-generated spread with default risk only and the incremental spread after including personal taxes, where the personal income tax rate  $\tau$  is linked to the effective equity tax rate  $\tau_E$  by the formula  $\tau_E = (1 - \delta)\alpha\tau + \delta\tau$  (see Engel et al. 1999 and Graham 2003) with  $\tau_E$  equal to 12% based on Graham (1999). The thinnest column includes the liquidity premium taken from de Jong and Driessen (2004). The curve with dots traces the average observed spreads on 10-year bonds of different ratings from Huang and Huang (2003).

income tax rates ( $\tau$ ) range from 17.4% to 40.7%, and the corresponding equity return tax rates ( $\tau_E$ ) range from 9.3% to 21.8%. The average implied equity tax rate is 11.7% for investment-grade bonds, which is almost the same as Graham's (1999) estimate for the equity tax rate. The average implied ordinary income tax rate for investment-grade bonds is about 22%, which is also very close to the 22.43% mark implied by the equity tax rate of 12% reported by Graham (1999). By contrast, the average implied income tax rate for junk bonds is 38%, which is relatively high compared to the estimates for investment-grade bonds. As shown in Figure 5, about 10% of the observed spread for junk bonds is left unexplained when we plug in  $\tau = 22.43\%$ . As expected, the implied income tax rate has to be higher to catch up with the observed junk bond spread when we calibrate the model. Although the implied tax rate seems high, it is below the maximum statutory personal income tax rate over the period (1973–1993) covered by our calibration. Furthermore, the implied tax rate for junk bonds is below the maximum rate of 48% and, in fact, is only slightly above the mean of 35% estimated by Graham (1999) for the tax rate of the marginal investor of government bonds over the period 1980–1994.<sup>13</sup> Thus, it might be

<sup>13</sup> See Graham (1999, p. 161). Graham's estimates are based on the difference between the one-year Treasury bill rate and the one-year prime grade munis.



premature to assert that the relatively high implied tax rate for junk bonds is because of missing factors. Notwithstanding this caveat, the model appears to explain the spreads of investment-grade bonds reasonably well.

## 5. Conclusions

In this paper, we incorporate the effects of personal taxes into the term structure model. The structural model provides an integrated framework that simultaneously deals with the issues of capital structure, default probability, and yield spreads. It offers a unified approach for analyzing the behavior of leverage, default risk, and yield spreads as the firm's asset value, risk, taxes, interest rate, payout ratio, and bankruptcy costs change. We find that personal taxes affect the optimal leverage level and the firm value in the presence of default risk and bankruptcy costs. Given the maturity of debt, the default boundary is determined endogenously, which sets the optimal level of debt. Personal taxes have an impact on the firm's leverage by affecting the debt value and default boundary. On the one hand, personal taxes lower default boundary, leverage, and default risk. On the other hand, personal taxes reduce the net payoff of bonds, and so investors require higher yields to compensate for the tax burden. These tax-related effects depend on the marginal investor's income tax rate and the interaction between default and personal taxes. In general, the negative effect of the tax penalty outweighs the positive effect of reduced default risk, resulting in a higher yield spread.

With the model calibrated to actual default experience, results show that ignoring personal taxes leads to considerable underestimation of yield spreads. The model without taxes explains only 31% to 41% of observed investment-grade bond spreads. By contrast, using more realistic tax rates documented in the literature, we find that the model explains 47% to 87% of these spreads. Results show that personal taxes account for a substantial proportion of the corporate-Treasury spread for high-grade bonds.

We further estimate the implied income tax rates by calibrating the model against the observed spreads adjusted for a reasonable amount of liquidity premium. We find that the implied personal income tax rates for investment grade bonds are very close to empirical tax rates. The implied income tax rates for junk bonds are higher but still within the range estimated by Graham (1999).

Finally, although personal taxes play an important role, liquidity risk should also be taken into account to better explain corporate bond spread. In this paper, we use an ad hoc approach to add the liquidity premium to the spread estimated by the structural model. This

approach ignores the interactive effect between default and liquidity risk. This effect is likely to be positive; that is, lower grade bonds tend to have lower liquidity (see Ericsson and Renault 2001). Ignoring this interactive default-liquidity effect may understate the corporate bond spread. Because this effect is potentially more important for junk bonds, it may explain why the model still underestimates the spreads of these bonds. Nevertheless, to fully account for this interactive default-liquidity effect, one would need to model liquidity risk explicitly. We leave this for future work.

An online supplement to this paper is available on the *Management Science* website (<http://mansci.pubs.informs.org/ecompanion.html>).

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## References

- Anderson, R., S. Sundaresan. 1996. Design and valuation of debt contracts. *Rev. Financial Stud.* **9** 37–68.
- Anderson, R., S. Sundaresan, P. Tychon. 1996. Strategic analysis of contingent claims. *Eur. Econom. Rev.* **40** 871–881.
- Andrade, G., S. Kaplan. 1998. How costly is financial (not economic) distress? Evidence from highly levered transactions that became distressed. *J. Finance* **53** 1443–1493.
- Benninga, S., O. H. Sarig. 1997. *Corporate Finance: A Valuation Approach*. McGraw-Hill, New York.
- Bhandari, C. 1988. Debt/equity ratio and expected common stock returns: Empirical evidence. *J. Finance* **43** 507–528.
- Black, F., J. Cox. 1976. Valuing corporate securities: Some effects of bond indenture provisions. *J. Finance* **31** 351–367.
- Black, F., M. Scholes. 1973. The pricing of options and corporate liabilities. *J. Political Econom.* **81** 637–659.
- Brealey, R. A., S. C. Myers. 1996. *Principles of Corporate Finance*, 5th ed. McGraw-Hill, New York.
- Brennan, M., E. Schwartz. 1978. Corporate income taxes, valuation, and the problem of optimal capital structure. *J. Bus.* **51** 103–114.
- Caouette, J. B., E. I. Altman, P. Narayanan. 1998. *Managing Credit Risk: The Next Great Financial Challenge*. John Wiley and Sons, New York.
- Collin-Dufresne, P., R. Goldstein. 2001. Do credit spreads reflect stationary leverage ratios? *J. Finance* **56** 1926–1957.
- Collin-Dufresne, P., R. S. Goldstein, J. Helwege. 2003. Is credit event risk priced? Modeling contagion via the updating beliefs. Working paper, Carnegie Mellon University, Pittsburgh, PA.
- Collin-Dufresne, P., R. S. Goldstein, J. S. Martin. 2001. The determinants of credit spread changes. *J. Finance* **56** 2177–2207.
- Constantinides, G. M., J. E. Ingersoll. 1984. Optimal bond trading with personal taxes. *J. Financial Econom.* **13** 299–335.
- Cooper, I. A., S. A. Davydenko. 2003. Using yield spreads to estimate expected returns on debt and equity. Working paper, London Business School, London, UK.

- de Jong, F., J. Driessen. 2004. Liquidity risk premia in corporate bond and equity markets. Working paper, University of Amsterdam, Amsterdam, The Netherlands.
- Duffie, J. D., D. Lando. 2001. Term structure of credit spreads with incomplete accounting information. *Econometrica* **69** 633–664.
- Elton, E. J., M. J. Gruber, D. Agrawal, C. Mann. 2001. Explaining the rate spread on corporate bonds. *J. Finance* **56** 247–277.
- Engel, E., M. Erickson, E. Maydew. 1999. Debt-equity hybrid securities. *J. Accounting Res.* **36** 249–274.
- Eom, Y. H., J. Helwege, J.-Z. Huang. 2003. Structural models of corporate bond pricing: An empirical analysis. *Rev. Financial Stud.* **17** 499–544.
- Ericsson, J., O. Renault. 2001. Liquidity and credit risk. Research paper series. International Center for Financial Asset Management and Engineering, Geneva, Switzerland.
- Fabozzi, F. J., D. Z. Nirenberg. 1991. Federal income tax treatment of fixed income securities. F. J. Fabozzi, ed. *Handbook of Fixed Income Securities*. Dow Jones-Irwin, Homewood, IL.
- Goldstein, R., N. Ju, H. Leland. 2001. An EBIT-based model of dynamic capital structure. *J. Bus.* **74** 483–512.
- Graham, J. R. 1999. Do personal taxes affect corporate financing decisions? *J. Public Econom.* **73** 147–185.
- Graham, J. R. 2000. How big are the tax benefits of debt? *J. Finance* **55** 1901–1941.
- Graham, J. R. 2003. Taxes and corporate finance: A review. *Rev. Financial Stud.* **16** 1075–1129.
- Harrison, J. M. 1990. *Brownian Motion and Stochastic Flow Systems*. Krieger Publishing Company, Malabar, FL.
- Huang, J.-Z., M. Huang. 2003. How much of the corporate-Treasury yield spread is due to credit risk? Working paper, Graduate School of Business, Stanford University, Stanford, CA.
- Huang, J.-Z., N. Ju, H. Ou-Yang. 2003. A model of optimal capital structure with stochastic interest rates. Working paper, Pennsylvania State University, University Park, PA.
- Jones, E. P., S. Mason, E. Rosenfeld. 1984. Contingent claims analysis of corporate capital structures: An empirical investigation. *J. Finance* **39** 611–625.
- Judd, K. L. 1998. *Numerical Methods in Economics*. MIT Press, Cambridge, MA.
- Kidwell, D., C. Trzcinka. 1982. Municipal bond pricing and the New York City fiscal crisis. *J. Finance* **37** 1239–1246.
- Kim, I. J., K. Ramaswamy, S. Sundaresan. 1993. Does default risk in coupons affect the valuation of corporate bonds? *Financial Management* **22** 117–131.
- Leland, H. 1994. Corporate debt value, bond covenants, and optional capital structure. *J. Finance* **49** 1213–1252.
- Leland, H. 1998. Agency costs, risk management, and capital structure. *J. Finance* **53** 1213–1242.
- Leland, H. 2002. Predictions of expected default frequencies in structural models of debt. Presentation, Venice Conference on Credit Risk, Venice, Italy.
- Leland, H., K. Toft. 1996. Optimal capital structure, endogenous bankruptcy, and the term structure of credit spreads. *J. Finance* **51** 987–1019.
- Liu, S., J. Shi, J. Wang, C. Wu. 2004. How much of the corporate bond spread is due to personal taxes? 2005 Western Finance Association Conference, Portland, OR.
- Longstaff, F., E. Schwartz. 1995. A simple approach to valuing risky fixed and floating rate debt. *J. Finance* **50** 789–891.
- Mella-Barral, P. W. Perraudin. 1997. Strategic debt service. *J. Finance* **52** 531–566.
- Merton, R. 1974. On the pricing of corporate debt: The risk structure of interest rates. *J. Finance* **29** 449–470.
- Miller, M. 1977. Debt and taxes. *J. Finance* **32** 261–275.
- Modigliani, F., M. Miller. 1958. The cost of capital, corporation finance, and the theory of investment. *Amer. Econom. Rev.* **48** 261–297.
- Modigliani, F., M. Miller. 1963. Corporate income taxes and the cost of capital: A correction. *Amer. Econom. Rev.* **53** 433–443.
- Rubinstein, M., E. Reiner. 1991. Breaking down the barriers. *Risk Magazine* **4** 28–35.
- Schaefer, S., I. Strebulaev. 2004. Structural models are useful: Evidence from hedge ratios. Working paper, London Business School, London, UK.
- Sick, G. A. 1990. Tax-adjusted discount rates. *Management Sci.* **36** 1432–1450.
- Strebulaev, I. 2004. Do capital structure tests mean what they say? Working paper, London Business School, London, UK.
- Taggart, R. 1991. Consistent valuation and cost of capital expressions with corporate and personal taxes. *Financial Management* **20** 8–20.
- Trzcinka, C. 1982. The pricing of tax-exempt bonds and the Miller hypothesis. *J. Finance* **37** 907–923.
- Yawitz, J. B., K. J. Maloney, L. H. Ederington. 1985. Taxes, default risk and yield spreads. *J. Finance* **40** 1127–1140.
- Zhou, C. 2001. The term structure of credit spreads with jump risk. *J. Banking Finance* **25** 2015–2040.